Laboratory of Digital Control Techniques

Exercise 3

Setting of digital PID controllers

I. Aims

1. Learning methods of setting digital PID controllers.
2. Performance analysis of various types of industrial digital controllers
3. Setting optimization (by-hand adjustments) of digital controllers.

II. Framework

1. Determine the static and dynamic parameters of the object $G_0(s)$ (before correction):

A
$$G_0(s)=\frac{(N/20)^2+(I/5)^2}{\left(\frac{1}{N}s+1\right)\cdot\left(s+N/20-j(I/5)\right)\cdot\left(s+N/20+j(I/5)\right)}$$

B
$$G_0(s)=\frac{(N/15)^2+(I/20)^2}{\left(\frac{1}{0{,}1\cdot N}s+1\right)\cdot\left(s+N/15-j(I/20)\right)\cdot\left(s+N/15+j(I/20)\right)}$$

– based on a unit step response of an object, choose an appropriate sampling frequency ($f_s \rightarrow T_s$),

2. Design a digital industrial controller (P, PI, PID type) for the given object, in a system as shown in Fig. 1 (see Appendix):

a) using an appropriate method, determine the coefficients K_p, K_i, K_d of a digital industrial controller $G_R(z)$:

– prepare a model of the control system in Simulink,

– examine the unit step response of the system after adjustment (determine the static and dynamic parameters),

– assess the quality of obtained regulation on the basis of regulation indicators - relations (3) to (6).

b) perform manual tuning of the coefficients Kp, Ki, Kd of the digital industrial controller $G_R(z)$ in order to optimize the static and dynamic parameters of the control system:

– prepare a model of the control system in Simulink,

– examine the unit step response of the system after correction (determine the static and dynamic parameters),

– assess the quality of obtained regulation on the basis of regulation indicators - relations (3) to (6).

– compare the operation with the system from point II.2a (with particular emphasis on dynamic parameters and control indicators).

3. Examine the robustness of the designed digital controllers to changes in the parameters of the control object:

– for this purpose, it is necessary to check the designed regulators (p. II.2b) for the object with the $G_0(s)$ transmittance, when the actual object transmittance $G'_0(s)$ differs from that adopted in the design process Fig. 7. (see Appendix).

III. Appendix

1. Assuming that the designed G_R(z) controller is to operate in the system shown in Fig. 1:

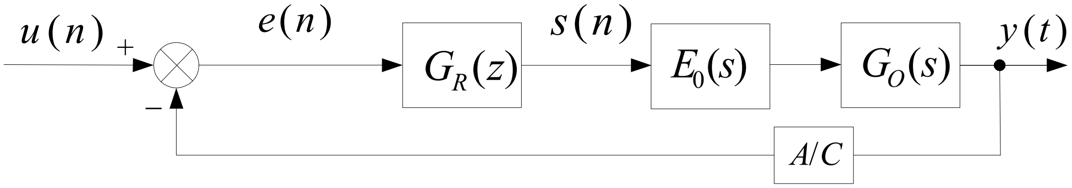


Fig. 1. Control system with negative feedback.

Let us assume that the transmittance of the digital industrial controller G_R(z) is given by the relation (1), while the block diagram is shown in Fig. 2.

$$G_{PID}(z) = \frac{S(z)}{E(z)} = K_p + K_i \frac{z+1}{z-1} + K_d \frac{z-1}{z} \tag{1}$$

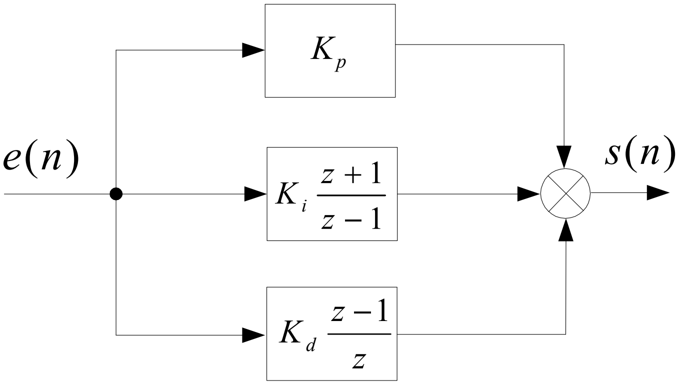


Fig. 2. Digital industrial PID controller.

Selection of settings for digital industrial controllers on the basis of a vibration test

This method of estimating the settings of industrial controllers is used for objects of at least third order and when the response of such an object to a unit step is oscillatory. The selection of the settings of industrial controllers on the basis of the vibration test consists in determining the value of the critical gain k_{gr} and the oscillation period T_{osc}. To do this, in the system shown in Fig. 3a. increase the gain K_p of the proportional part of the PID controller (the other parts are then inactive) so that when forced by a unit step, the system is brought to the stability limit, see Fig. 3b. Then the critical gain will be equal to the current value of the proportional gain $k_{gr} = K_p$, and the vibration period T_{osc} can be estimated as shown in Fig. 3b. Next, you should define the settings of digital industrial controllers (type P, PI, PID) according to Table 1.

Hint: *The frequency characteristics of the considered control object plotted on the Nichols card can be used to roughly determine the value of the critical gain (before proceeding to the oscillation test).*

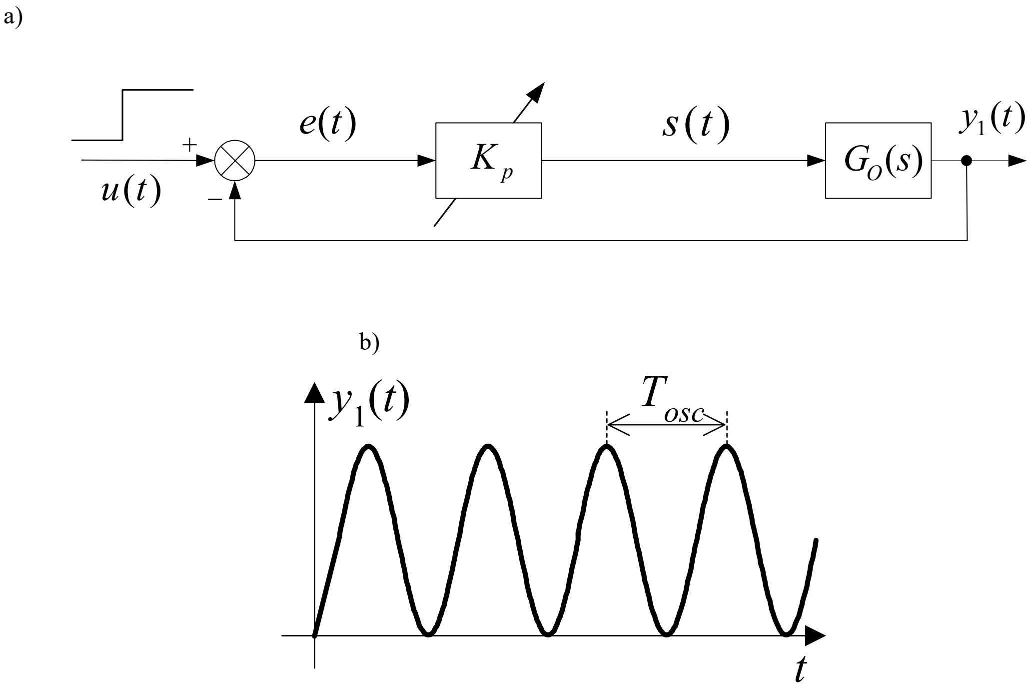


Fig. 3. Estimating the parameters of industrial controllers based on the oscillation test: a) system diagram, b) unit step response at the limit of stability.

Table 1. Settings of digital industrial controllers for oscillation test data.

Regulator type	Settings of individual parameters		
	K_p	K_i	K_d
P	$0{,}5k_{gr}$	0	0
PI	$0{,}45k_{gr}$	$\frac{0{,}6 \cdot K_p \cdot T_p}{T_{osc}}$	0
PID	$0{,}6k_{gr}$	$\frac{K_p \cdot T_p}{T_{osc}}$	$\frac{K_p \cdot T_{osc}}{8 \cdot T_p}$

Selection of settings for digital industrial controllers based on the response to the unit step of the control object

If the object - regardless of its order - has an inertial response to a unit step (see Fig. 4), then its model can be approximated by the following transfer function:

$$G_{AO}(s) = \frac{k \cdot e^{-sT_0}}{Ts + 1} \tag{2}$$

With such a simplifying assumption, it is possible to determine the settings of digital industrial PID controllers on the basis of parameters k, T_0 and T (read from the response to the unit step of the object in an open circuit - Fig. 4), see Table 2.

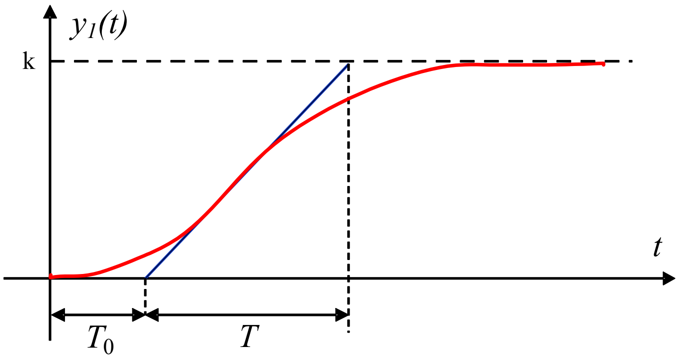


Fig. 4. Response to the unit step of an inertial object.

Table 2. Digital industrial controller settings for test data in an open circuit.

Regulator type	Settings of individual parameters		
	K_p	K_i	K_d
P	$\frac{T}{k \cdot T_0}$	0	0
PI	$\frac{0{,}586}{k}\left(\frac{T}{T_0}\right)^{0{,}916}$	$\frac{K_p \cdot T_p \cdot \left(1{,}03 - 0{,}165\frac{T_0}{T}\right)}{2T}$	0
PID	$\frac{0{,}965}{k}\left(\frac{T}{T_0}\right)^{0{,}855}$	$\frac{K_p \cdot T_p \cdot \left(0{,}796 - 0{,}147\frac{T_0}{T}\right)}{2T}$	$\frac{K_p \cdot 0{,}308 \cdot T \cdot \left(\frac{T_0}{T}\right)^{0{,}929}}{T_p}$

For the designed regulators it is necessary to optimize obtained settings (manual tuning) in order to obtain the improvement of the regulation indicators. The influence of individual digital controller settings on the static and dynamic parameters of the object is presented in Table 3.

Table 3. Influence of the controller settings on the parameters of the closed-loop object.

Adjusted parameter	Static and dynamic parameters			
	T_r	y_p	t_u	Δy
K_p ↗	decreasing	increasing	-	decreasing
K_i ↗	decreasing	increasing	increasing	eliminates
K_d ↗	-	decreasing	decreasing	-

Regulatory quality indicators

1. Integral criteria are most often used to assess the quality of closed loop control. For the control error to be the smallest, the integral of the error $\int_0^\infty e(t)\,\mathrm{d}t$ must approach zero (Fig. 5.).

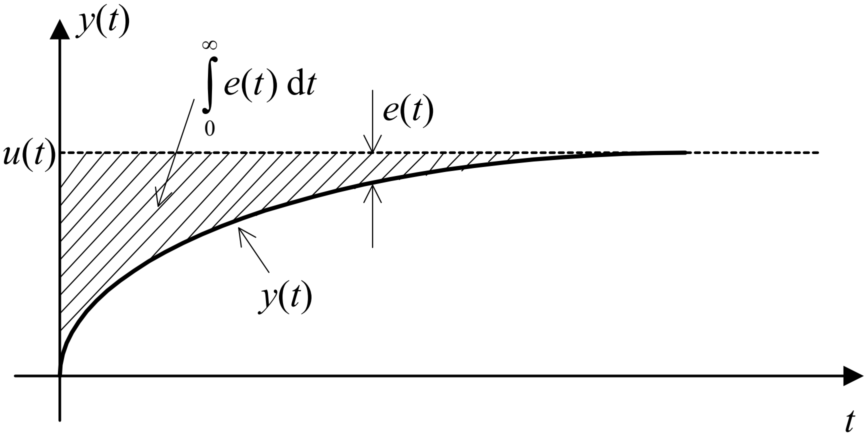


Fig. 5. Response to the unit step of a closed system - interpretation of the integral criterion.

In practice, several versions of the integral criterion are used to assess the quality of regulation. Below, the most important of them are listed (these should be used to assess the quality of regulation of the designed regulators):

- Integral Squared Error:

$$I = \int_0^\infty (e(t))^2\,dt \tag{3}$$

- Integral of Time multiplied by Squared Error:

$$I_1 = \int_0^\infty t(e(t))^2\,dt \tag{4}$$

- Integral of Absolute value of Error:

$$I_2 = \int_0^\infty |e(t)|\,dt \tag{5}$$

- Integral of Time multiplied by Absolute value of Error

$$I_3 = \int_0^\infty t|e(t)|\,dt \tag{6}$$

The above integral criteria (equations (3) - (6)) in a discrete system can be calculated using the algorithm of numerical integration using the trapezoidal method (see Fig. 6).

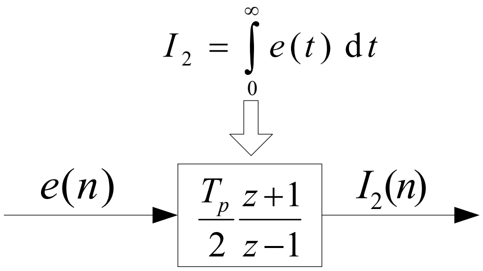


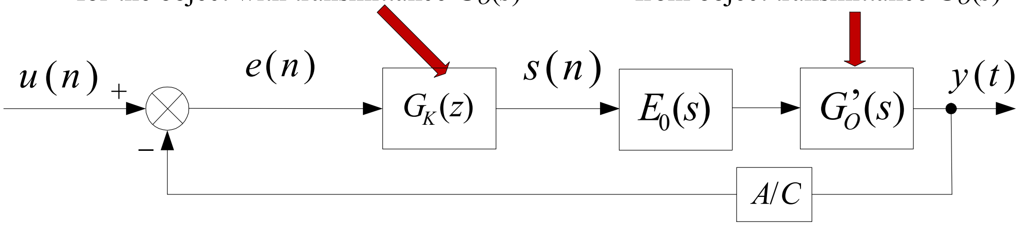
Fig. 6. Numerical integration using the trapezoidal method.

2. Regulator robustness test.

The robustness of the regulator means the tolerance for errors occurring during identification (incorrect model structure or approximation of the transmittance of the control object) or for changes in the object parameters (amplification factor, time constants, delay) during its operation.

It is desirable that even if the mathematical model of the object adopted in the design process is not correct, the control system will be stable and its control close to optimal.

Therefore, in order to test the robustness of the regulator, the regulator should be designed for an object with the $G_0(s)$ transmittance (as described in points II 3b), and then its operation should be checked when the actual transmittance of the $G'_0(s)$ object differs from that which was adopted in the design process, see Fig. 7.


Transfer function of the controller designed for the object with transmittance $G_O(s)$

Object transmittance slightly different from object transmittance $G_O(s)$

Fig. 7. Diagram of the circuit for the analysis of resistance of the designed correctors.

3. Useful Matlab commands.

When designing regulators, you can use the following commands in Matlab:

c2dm
feedback
series
ginput
nichols
help