Laboratory of Digital Control Techniques

Exercise 5

Designing a modal equalizer

I. Purpose of the exercise

1. Learning the principles of designing digital modal (state) controllers.
2. Analysis of the operation of the modal corrector in a system in which all state variables are available for measurement.
3. Analysis of the operation of the modal corrector in a system in which the state variables are not available for measurement.
4. Testing the immunity of regulators referred to in points I.2 and I.3.

II. Framework exercise program

1. Determine the static and dynamic parameters of the object $G_0(s)$ (before correction):

AND $$G_0(s) = \frac{1}{(Ns+1)\cdot(Is+1)}$$

B $$G_0(s) = \frac{N^2 + (2\cdot I)^2}{(s+N-j2\cdot I)\cdot(s+N+j2\cdot I)}$$

– based on the response to a unit jump of a given object, select the appropriate sampling frequency.

2. Determine the values of the gain matrix from state variables of the modal equalizer ***K*** operating in a system in which all state variables are available, see Fig. 1. (**Appendix**).

– before starting design, the controllability of the given object should be checked,

– the selected corrector should improve the selected dynamic parameters of the system in the manner specified by the student,

– create a model of the control system in Simulink,

– examine the response to a unit jump of the system after correction,

– scale the set signal appropriately so that in the steady state the control error is 0,

– assess the quality of regulation and discuss the 'intensity' of control.

3. Determine the values of the gain matrix from state variables of the modal equalizer ***K*** operating in a system in which all state variables are not available, see Fig. 2. (**Appendix**).

– calculate the digital state model of the control object [$\boldsymbol{A}_D$, $\boldsymbol{B}_D$, $\boldsymbol{C}_D$, $\boldsymbol{D}_D$] taking into account sampling and extrapolation (assume zero-order extrapolation),

– before starting design, the controllability of the given object should be checked,

– the selected corrector should improve the selected dynamic parameters of the system in the manner assumed by the student (the same assumptions can be made as in point II.2),

– create a model of the control system in Simulink,

– examine the response to a unit jump of the system after correction,

– scale the set signal appropriately so that in the steady state the control error is 0,

– assess the quality of regulation and discuss the 'intensity' of control,

– compare the operation with the system from point II.2 (with particular emphasis on dynamic parameters and control signals in both systems).

4. Test the resistance of the correctors from points II.2 and II.3 to changes in the parameters of the control object.
   – design correctors for an object with transmittance $G_0(s)$ (as described above),
   – check the operation of correctors when the actual transmittance of the object $G'_0(s)$ differs from that assumed in the design process,
   – compare the operation of both solutions (from point II.2 and II.3).

III. Addition

1. Designing modal equalizers.

Modal control is based on the design of systems with feedback (via an appropriate gain matrix) from state variables (Fig. 1), which allows the poles of the closed system to be located in places specified by the designer. This - - appropriate arrangement of poles - as we know, guarantees obtaining a system with the desired static and dynamic properties.

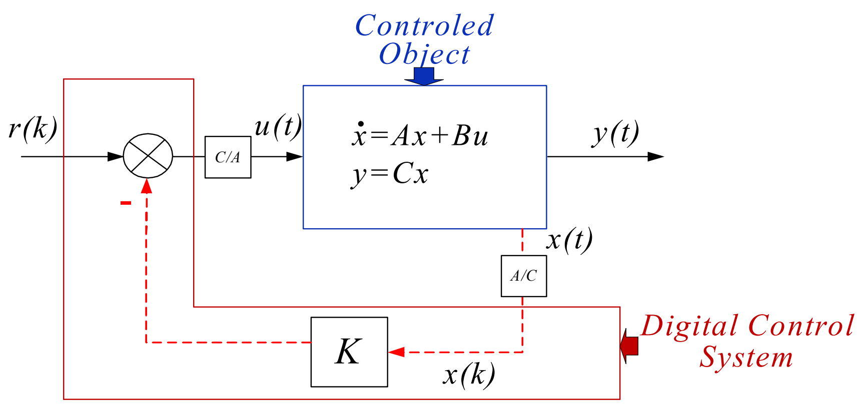

Fig. 1. Digital control system using a modal equalizer.

We consider a linear control system that can be described as follows:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)\\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t)\end{aligned} \tag{1}$$

In order to be able to design a modal equalizer for a given object, the object must be controllable . The Kalman criterion says that the system is controllable if and only if the order of the Kalman matrix

$$\Omega = \left[\mathbf{B}, \mathbf{AB}, \mathbf{A}^2\mathbf{B}, ..., \mathbf{A}^{n-1}\mathbf{B}\right] \tag{2}$$

is full, i.e.

$$rank(\mathbf{\Omega}) = n \tag{3}$$

where n is the row of matrix $\boldsymbol{A}$ (row of the control object).

In the case of a one-dimensional system, the following criterion can be used

$$\det\left|\mathbf{B} \quad \mathbf{AB} \quad ... \quad \mathbf{A}^{n-1}\mathbf{B}\right| \neq 0 \tag{4}$$

As mentioned above , the $\boldsymbol{K}$ matrix is introduced into the control system , which is responsible for the appropriate location of the poles of the closed system. With this assumption, the transmittance of the closed system is:

$$K(s) = \mathbf{C}[s\mathbf{I} - \mathbf{F}]^{-1}\mathbf{B} = \mathbf{C}\frac{adj[s\mathbf{I} - \mathbf{F}]}{\det([s\mathbf{I} - \mathbf{F}])}\mathbf{B} = \frac{H(s)}{T(s)} \quad (5)$$

Assuming that

$$\mathbf{F} = \mathbf{A} - \mathbf{BK} \quad (6)$$

The characteristic polynomial of a closed system can be determined as follows

$$T(s) = \det\left[s\mathbf{I} - (\mathbf{A} - \mathbf{BK})\right] = \alpha_n s^n + \alpha_{n-1}s^{n-1} + ... + \alpha_0 = (s + b_1)(s + b_2)...(s + b_n) \quad (7)$$

where both α_{and} (coefficients of the characteristic polynomial of the system after correction), as well as b_j (the poles of the system after applying the state corrector) are known quantities, selected by the designer (more on this topic can be found in the instructions for Exercise 4).

To improve the dynamics, the poles of the control system after applying the controller ([$b_1, b_2, \ldots, b_n$]) should be located in appropriate places in relation to the poles of the system before correction. The choice of these poles is decided by the designer.

Once the poles of the system are determined after applying the modal corrector, the problem of finding the appropriate matrix of gains from state variables ***K*** comes down to solving equation (7). This task can be complex (especially in the case of higher order systems), so for this purpose you can use the Matlab program, in which the ***K matrix*** can be calculated as follows:

$$\mathbf{K} = place(\mathbf{A}, \mathbf{B}, [b_1\ b_2\ b_3\ .....\ b_n]) \quad (8)$$

Because in a system with a modal equalizer we do not compare the set signal with the output signal; and instead, we multiply the state vector times the ***K matrix*** and subtract the result from the set value. This means that in the steady state the output value may differ significantly from the set value. To achieve the desired effect, the reference signal $r(k)$ must be appropriately scaled using gain Ns , see Fig. 2.

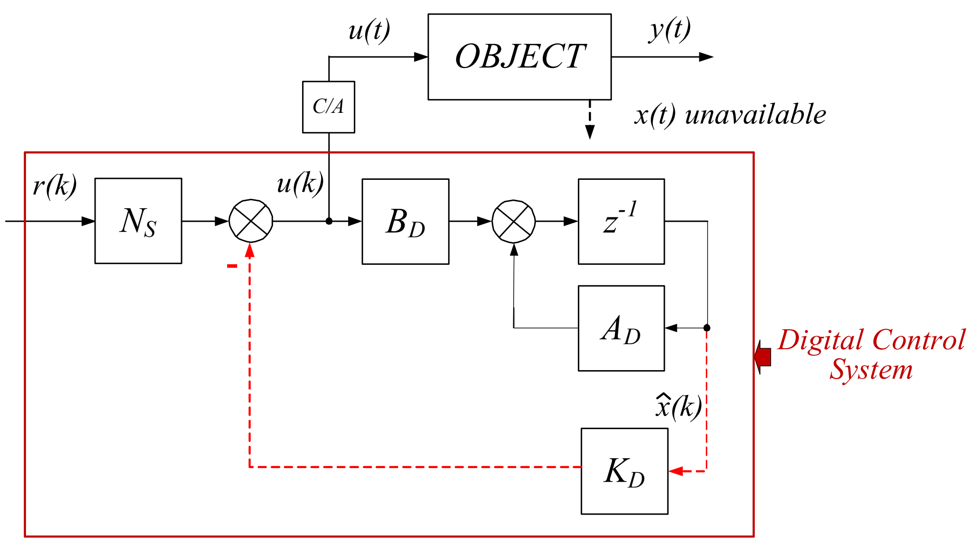

Fig. 2. Digital control system using a modal corrector - when the state variables of the control object are not available for measurement.

The above considerations apply to the situation in which we have access to all state variables. If we would like to implement control using a modal equalizer in a system in which we cannot measure the state variables, we can use a solution in which the state variables are estimated. Fig. 2 shows a system where state variables are estimated using a digital state model.

With this approach, design should begin by determining a digital state model of the control object [$\boldsymbol{A_D}, \boldsymbol{B_D}, \boldsymbol{C_D}, \boldsymbol{D_D}$]. Further steps are identical to those described for the system with available state variables (see the beginning of this section).

2. Useful commands.

When designing correctors, you can use the following commands available in Matlab:

c2dm
feedback
series
zgrid
ginput
place
help