Laboratory of Digital Control Techniques

Exercise 6

Control using modal controllers with a state observer

I. Purpose of the exercise

1. Learning the principles of designing and selecting parameters of state observers cooperating with a digital modal (state) controller.
2. Analysis of the properties of the designed system in various operating conditions and testing of its resistance to changes in the parameters of the control object.

II. Framework exercise program

1. Determine the static and dynamic parameters of the object $G_0(s)$ (before correction):

A
$$G_0(s) = \frac{1}{(Ns+1)\cdot(Is+1)}$$

B
$$G_0(s) = \frac{N^2 + (2\cdot I)^2}{(s+N-j2\cdot I)\cdot(s+N+j2\cdot I)}$$

– based on the response to a unit jump of a given object, select the appropriate sampling frequency.

2. Design a modal equalizer for a system in which state variables are not available. To estimate state variables, use a state observer working in the system as in Fig. 1.

– calculate the digital state model of the control object [$\boldsymbol{A}_D$, $\boldsymbol{B}_D$, $\boldsymbol{C}_D$, $\boldsymbol{D}_D$] taking into account sampling and extrapolation (assume zero-order extrapolation),

– before starting design, the controllability and observability of the given object should be checked,

– determine the gain matrix of the state equalizer $\boldsymbol{K}$, which will improve the selected dynamic parameters of the system,

$\boldsymbol{L}$ according to the previously designed modal corrector ,

– create a model of the control system in Simulink,

– examine the response to a unit jump of the system after correction,

– scale the set signal appropriately so that in the steady state the control error is 0,

– assess the quality of regulation,

– compare the operation of the designed equalizer with the system from Exercise 5, point II.3 (with particular emphasis on the situation when the actual transmittance of the object $G'_0(s)$ differs from that assumed in the design process).

III. Addition

1. Designing modal equalizers with a state observer.

Modal control requires the availability of state variables of the control object, as described in Exercise 5. However, individual state variables are often not available to the control system. Therefore, it is necessary to construct state observers whose task is to estimate the internal state of the control object as precisely as possible. For this purpose, the following observer structure can be proposed (according to Luenberger), where $\boldsymbol{L}_D$ is the observer's gain matrix, see Fig. 1.

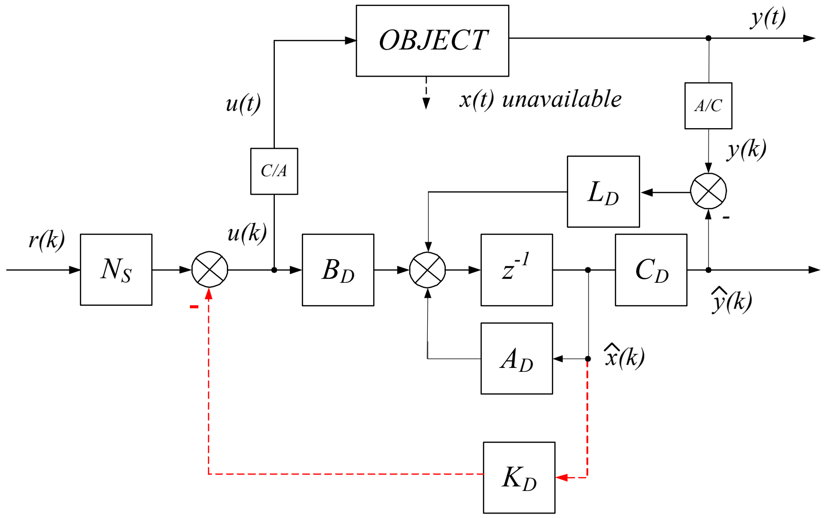

Fig. 1. Digital control system using a modal equalizer and a state observer.

For this purpose, let's consider a linear discrete object with a stateful equalizer:

$$\begin{aligned}\mathbf{x}[(k+1)] &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k)\end{aligned} \tag{1}$$

for which the controller equation is as follows:

$$\mathbf{u}(k) = \mathbf{r}(k) - \mathbf{K}_D\hat{\mathbf{x}}(k) \tag{2}$$

Where:
$\boldsymbol{K}_D$ – is the gain matrix from the modal equalizer state variables,
$r(k)$ – set value signal,
$\hat{x}(k)$ – estimated vector of state variables.
Then the state vector estimation error takes the following form

$$\mathbf{e}(k) = \mathbf{x}(k) - \hat{\mathbf{x}}(k) \tag{3}$$

and the state observer equations:

$$\begin{aligned}\hat{\mathbf{x}}[(k+1)] &= \mathbf{A}\hat{\mathbf{x}}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{L}_\mathbf{D}[\mathbf{y}(k) - \hat{\mathbf{y}}(k)] \\ \hat{\mathbf{y}}(k) &= \mathbf{C}\hat{\mathbf{x}}(k)\end{aligned} \tag{4}$$

Where:
$\hat{y}(k)$ – estimated output vector.

By rearranging equations (1), (3) and (4) respectively, we obtain the following equation:

$$\mathbf{e}[(k+1)] = \mathbf{A}\mathbf{e}(k) - \mathbf{L}_\mathbf{D}[\mathbf{y}(k) - \hat{\mathbf{y}}(k)] \tag{5}$$

which ultimately leads to:

$$\mathbf{e}[(k+1)] = (\mathbf{A} - \mathbf{L}_\mathbf{D}\mathbf{C})\mathbf{e}(k) = \mathbf{F}_0\mathbf{e}(k) \tag{6}$$

Where:

$$\mathbf{F}_\mathbf{0} = \mathbf{A} - \mathbf{L}_\mathbf{D}\mathbf{C} \tag{7}$$

Therefore, to design a state observer, you must first define the $\boldsymbol{F}_{0\ matrix}$, which, according to equation (7), determines the values of the gain matrix of the sought observer. We require the state observer to guarantee quick convergence of the state variable estimation process and the stability of the control system (the estimation error (6) should approach zero). Therefore, according to equation (6), matrix $\boldsymbol{F}_0$ must have all eigenvalues $[z_1, z_2, \ldots, z_n]$ inside the unit circle in the *z plane* . Additionally, it should be taken into account that the observer's own values should provide him with a much faster response (usually several times) compared to the state regulator. When designing a state observer, you must remember this.

L D matrix is determined in a similar way as for the feedback matrix from the state variables of the modal equalizer (see Exercise 5). Recall that for the modal equalizer we had:

$$\mathbf{F} = \mathbf{A} - \mathbf{BK} \tag{8}$$

and

$$\det([z\mathbf{I} - \mathbf{F}]) = \det([z\mathbf{I} - (\mathbf{A} - \mathbf{BK})]) = (z - b_1)(z - b_2)........(z - b_n) \tag{9}$$

While for the observer we get:

$$\mathbf{F}_0 = \mathbf{A} - \mathbf{L_D C} \tag{10}$$

and

$$\det([z\mathbf{I} - \mathbf{F}_o]) = \det([z\mathbf{I} - (\mathbf{A} - \mathbf{L_D C})]) = (z - z_1)(z - z_2)........(z - z_n) \tag{11}$$

However, in the case of the state observer we must (due to the dimensions of the matrix) use transposed matrices:

$$\det([z\mathbf{I} - (\mathbf{A} - \mathbf{L_D C})]) = \det([z\mathbf{I} - (\mathbf{A}^T - \mathbf{C}^T \mathbf{L_D^T})]) \tag{12}$$

, you can determine the desired observer matrix ***LD*** using - similarly to the modal corrector - one of the Matlab functions:

$$\mathbf{L_D} = place(\mathbf{A}', \mathbf{C}', [z_1 \ z_2 \ .... \ z_n])' \tag{13}$$

The process of designing a modal equalizer with a state observer described above makes sense only if the control object is controllable (see Exercise 5) and observable. A linear stationary system with one output is observable if and only if the row of the observability matrix

$$\Omega = \begin{bmatrix} C \\ C \cdot \mathbf{A} \\ C \cdot \mathbf{A}^2 \\ \cdots \\ C \cdot \mathbf{A}^{n-1} \end{bmatrix} \tag{14}$$

is full,

$$rank(\Omega) = n \tag{15}$$

or when

$$\det(\Omega) \neq 0 \tag{16}$$

Both conditions (controllability and observability) must be checked before designing the state observer.

2 . Useful commands.

c2dm
feedback
series
zgrid
ginput
place
help