Laboratory of Digital Control Techniques

<u>Excercise 1</u>

Designing and testing the properties of filters with infinite impulse response

I. Aims

1. Learning the methods of designing digital filters with infinite impulse response.
2. Synthesis and implementation of a digital filter with given properties.
3. Analysis of the properties of the designed filter.

II. Exercise framework

1. Determine the transmittance of the digital recursive filter in the following form:

$$G(z) = \frac{\sum_{k=0}^{N-1} a(k) z^{-k}}{1 + \sum_{k=1}^{M} b(k) z^{-k}}$$

– in the design process, use the method consisting in the discretization of the analog prototype through bilinear transformation.
– the designed filter should be a:

Odd groups - low pass (LP),

Even groups - high pass (HP).

- cut-off frequency of the designed digital filter: fgc = (200 + ('group number') * 50) Hz.

- use the transfer function of 'group number' as a prototype (see chapter III).

- take the sampling frequency equal to: fs = (900 + 'group number' * 100) Hz.

Note!
Assume that the cut-off frequency is the frequency at which the filter amplification drops to -3dB.

2. Using the Matlab / Simulink environment:

   - remove the frequency characteristics of the obtained filter (assess whether the obtained filter corresponds to the above design assumptions),

   - examine the time responses of the filter for different input signals (perform spectral analysis of the signals before and after filtration). When selecting the frequency of the input signals, consider the shape of the obtained frequency characteristic of the filter obtained.

3. Perform the synthesis and analysis of the digital filter - as described in the above two points - when the analog prototype will be a higher order filter (e.g. third). Compare the properties with the lower order filter.

III Additional information

When designing a filter, the following transformations should be used:

$LP: \quad G(z) = G(s)_{|s \to A\frac{1-z^{-1}}{1+z^{-1}}}, \qquad A = \omega_{ga} ctg(\omega_{gc} T_p/2)$

$HP: \quad G(z) = G(s)_{|s \to B\frac{1+z^{-1}}{1-z^{-1}}}, \qquad B = \omega_{ga} tg(\omega_{gc} T_p/2)$

where:
G(z) - transmittance of the searched digital NOI filter,
G(s) - transmittance of the analog prototype,
ω_{ga} – cut-off pulsation of the filter *G(s),*
ω_{gc} – cut-off pulsation of the designed digital filter *G(z)*,

Standard filter transmittances:

1. Butterworth 2-order $G(s) = \dfrac{1}{s^2 + \sqrt{2}s + 1}$
2. Bessel 2-order $G(s) = \dfrac{1}{s^2 + 1.73s + 1}$
3. Tschebyschev I, 2-order $G(s) = \dfrac{1.43}{s^2 + 1.4256s + 1.5162}$
4. Butterworth 2-order $G(s) = \dfrac{1}{s^2 + \sqrt{2}s + 1}$
5. Bessel 2-order $G(s) = \dfrac{1}{s^2 + 1.73s + 1}$
6. Tschebyschev I, 2-order $G(s) = \dfrac{1.43}{s^2 + 1.4256s + 1.5162}$
7. Butterworth 2-order $G(s) = \dfrac{1}{s^2 + \sqrt{2}s + 1}$
8. Bessel 2-order $G(s) = \dfrac{1}{s^2 + 1.73s + 1}$
9. Tschebyschev I, 2-order $G(s) = \dfrac{1.43}{s^2 + 1.4256s + 1.5162}$

Butterworth 3-order $G(s) = \dfrac{1}{s^3 + 2s^2 + 2s + 1}$

Bessel 3-order $G(s) = \dfrac{1}{s^3 + 2{,}43s^2 + 2{,}47s + 1}$

Tschebyschev I, 3-order $G(s) = \dfrac{0{,}7157}{s^3 + 1{,}253s^2 + 1{,}535s + 0{,}7157}$