Laboratory of Digital Control Techniques

Exercise 4

Designing non-resistant and resistant digital equalizers

I. Purpose of the exercise

1. Learning the principles of designing digital controllers dedicated to a given facility.
2. Designing correctors that are not resistant to given dynamic parameters of the system after correction.
3. Designing correctors that are resistant to given dynamic parameters of the system after correction.
4. Comparison of the properties of resistant and non-resistant filters.

II. Framework exercise program

1. Determine the static and dynamic parameters of the object $G_0(s)$ (before correction):

AND $$G_0(s) = \frac{1}{(Ns+1)\cdot(Is+1)}$$

B $$G_0(s) = \frac{N^2 + (2\cdot I)^2}{(s+N-j2\cdot I)\cdot(s+N+j2\cdot I)}$$

– based on the response to a unit jump of a given object, select the appropriate sampling frequency (f_p),

2. Calculate the digital equivalent of the transmittance of the control object $G_{0E}(z)$ taking into account sampling and extrapolation (assume zero-order extrapolation).

3. Design a corrector that is not resistant to a given object, in the system as in Fig. 1 (**Appendix**):

a) determine the transmittance of the corrector $G_K(z)$ assuming that the system after applying the corrector $K(z)$ is to be minimal in time (assess the feasibility of the system),

– create a model of the control system in Simulink,

– examine the response to a unit jump of the system after correction,

– assess the quality of regulation and discuss the 'intensity' of control ($s(n)$).

b) determine the transmittance of the corrector $G_K(z)$ assuming that the response of the system after applying the corrector $K(z)$ is inertial in nature with a settling time three times shorter than that observed for the object before the correction $G_0(s)$ (the starting point for determining $K(z)$ may be the transmittance of a continuous object),

– create a model of the control system in Simulink,

– examine the response to a unit jump of the system after correction,

– assess the quality of regulation and discuss the 'intensity' of control ($s(n)$),

– compare the operation with the system from point II.3a (with particular emphasis on dynamic parameters and control signals in both systems).

c) determine the transmittance of the corrector $G_K(z)$ by specifying the transmittance poles $K(z)$ in such a way as to achieve the assumed dynamic parameters of the system after correction (appropriate overshoot and settling time):

– create a model of the control system in Simulink,

– examine the response to a unit jump of the system after correction,

– assess the quality of regulation and discuss the 'intensity' of control ($s(n)$),

– compare the operation with the systems from points II.3a and II.3b (with particular emphasis on dynamic parameters and control signals in these systems).

4. Design a corrector resistant to a given object, in the arrangement as in Fig. 1:

a) determine the transmittance of the corrector $G_K(z)$ assuming that the system after applying the corrector $K(z)$ is to be minimal in time (assess the feasibility of the system),

– create a model of the control system in Simulink,

– examine the response to a unit jump of the system after correction,

– assess the quality of regulation and discuss the 'intensity' of control ($s(n)$).

– test the corrector's resistance to changes in the parameters of the control object: for this purpose, you need to design a corrector for an object with transmittance $G_0(s)$ (as described above) and then check its operation in a situation where the actual transmittance of the object $G'_0(s)$ differs from the one adopted in the design process (Fig. 3 - **Addition**).
Carry out a similar test for the concealer from point 3a. Compare the results.

III. Addition

1. Designing a non-resistant concealer.

We assume that the designed $G_K(z)$ corrector is to work in the system shown in Fig. 1.

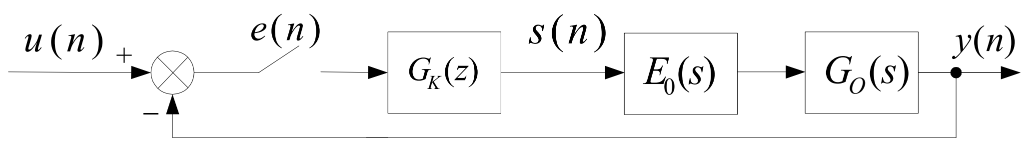

Fig. 1. Regulation system with negative feedback.

If the designed corrector is to be dedicated to an object with transmittance $G_0(s)$, its transmittance can be calculated according to the following relationship:

$$G_K(z) = \frac{K(z)}{1-K(z)} \cdot \frac{1}{G_{OE}(z)} \tag{1}$$

Where:
$G_{0E}(z)$ – digital equivalent of the transmittance of the control object $G_0(s)$ including sampling and extrapolation,
$K(z)$ – transmittance of the closed system assumed in the design process (from Fig. 1) after applying the *G corrector* $_K(z)$.

When selecting the transmittance $K(z)$, remember that for the correction to be possible, the condition must be met

$$n_t - n_h \geq n_a - n_b \tag{2}$$

Where:
n_b – degree of the polynomial of the transmittance numerator of the object $G_{0E}(z)$,
n_a – degree of the polynomial of the denominator of the object's transmittance $G_{0E}(z)$,
n_h – degree of the polynomial of the transmittance numerator of the closed system after correction $K(z)$,

n_t – degree of the polynomial of the denominator of the transmittance of the closed system after correction $K(z)$.

When designing a digital equalizer by specifying the poles of the system after correction (see point II.4c), you can use the characteristic shown in Fig. 2 (you can generate it in Matlab using the '*zgrid*' command).

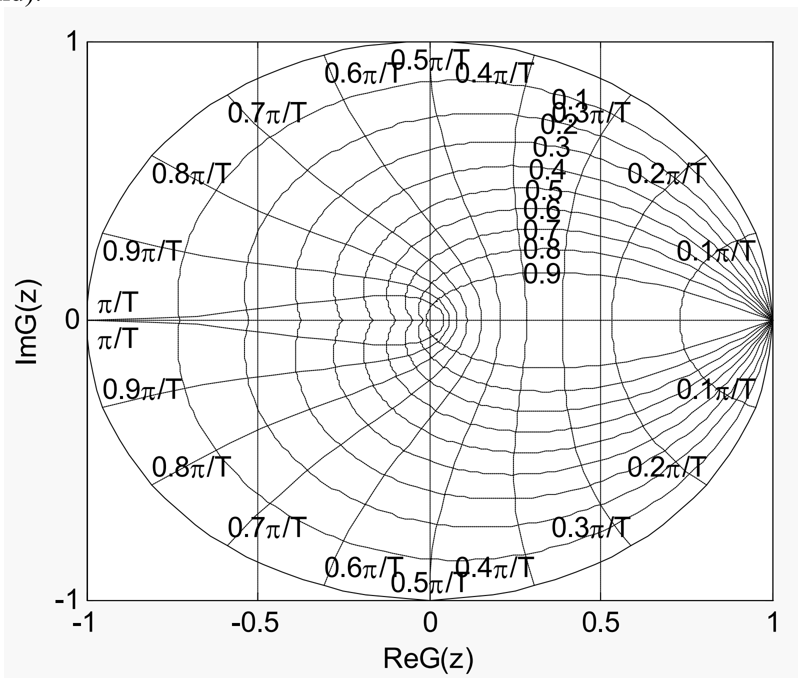

Fig. 2. Curves of constant damping coefficient (n) and constant natural pulsation (ω_n) on the z plane.

The figure above illustrates how the arrangement of the poles affects the values of the damping coefficient (n) and the system's own pulsation (ω_n). These two quantities, in turn, determine the overshoot values

$$y_p = 100\exp\left(\frac{-\pi n}{\sqrt{1-n^2}}\right) \tag{3}$$

and determination time

$$t_{u2\%} = 4\frac{1}{n\omega_n} \tag{4}$$

2. Designing a resistant concealer.

We assume that the designed $G_K(z)$ corrector is to work in the system shown in Fig. 1. Then the procedure for designing a resistant corrector is as follows:

STEP 1. Based on the realizability conditions (5), determine the degrees of the polynomials $T(z)$, $G(z)$ and $F(z)$ appearing in the synthesis equation.

$$\left.\begin{aligned} n_g &= n_a - 1 \\ n_f &\geq n_g \\ n_t &\geq n_a + n_f \end{aligned}\right\} \tag{5}$$

Where:
n_b – degree of the polynomial of the transmittance numerator of the object $G_{0E}(z)$,
n_a – degree of the polynomial of the denominator of the object's transmittance $G_{0E}(z)$,
n_g – degree of the polynomial of the numerator of the sought transmittance of the corrector $G_K(z)$,

n_f – degree of the polynomial of the denominator of the sought transmittance of the corrector $G_K(z)$,

n_t – degree of the polynomial of the denominator of the transmittance of the closed system after correction $K(z)$.

and

$$G_k(z) = \frac{G(z)}{F(z)} \tag{6}$$

$$G_{OE}(z) = \frac{B(z)}{A(z)} \tag{7}$$

$$K(z) = B(z)\frac{G(z)}{T(z)} \tag{8}$$

The conditions defined in equation (5) can be simplified if we assume that the designed equalizer is to be of the lowest possible order:

$$\left.\begin{aligned} n_g &= n_a - 1 \\ n_f &= \max(n_g, n_b) \\ n_t &= n_a + n_f \end{aligned}\right\} \tag{9}$$

This way you can determine the general formula of the transmittance of the corrector you are looking for:

$$G_k(z) = \frac{g_{n_g} z^{n_g} + g_{n_g-1} z^{n_g-1} + \ldots + g_1 z + g_0}{z^{n_f} + f_{n_f-1} z^{n_f-1} + \ldots + f_1 z + f_0} = \frac{G(z)}{F(z)} \tag{10}$$

STEP 2. Depending on the desired properties of the closed system after correction, the denominator $T(z)$ of the transmittance $K(z)$ should be set (the zeros of this polynomial determine the dynamics of the system after correction):

$$T(z) = t_{n_t} z^{n_t} + t_{n_t-1} z^{n_t-1} + \ldots + t_1 z + t_0 \tag{11}$$

STEP 3. We calculate the coefficients of the polynomials $G(z)$ and $F(z)$ by solving the polynomial synthesis equation

$$A(z)F(z) + B(z)G(z) = T(z) \tag{12}$$

The above equation, after arranging and comparing the coefficients side by side, can be written in matrix form:

$$\begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,ng+nf+1} \\ x_{1,1} & x_{2,2} & \cdots & x_{2,ng+nf+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_{ng+nf+1,1} & x_{ng+nf+1,2} & \cdots & x_{ng+nf+1,ng+nf+1} \end{bmatrix} \cdot \begin{bmatrix} f_{n_f-1} \\ \vdots \\ f_0 \\ g_{n_g} \\ \vdots \\ g_0 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{ng+nf+1} \end{bmatrix} \tag{13}$$

where x and y are known.

Therefore, the sought transmittance coefficients of the corrector can be calculated as follows (Matlab can be used for this purpose):

$$\begin{bmatrix} f_{n_f-1} \\ \vdots \\ f_0 \\ g_{n_g} \\ \vdots \\ g_0 \end{bmatrix} = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,ng+nf+1} \\ x_{1,1} & x_{2,2} & \cdots & x_{2,ng+nf+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_{ng+nf+1,1} & x_{ng+nf+1,2} & \cdots & x_{ng+nf+1,ng+nf+1} \end{bmatrix}^{-1} \cdot \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{ng+nf+1} \end{bmatrix} \quad (14)$$

3. Corrector resistance test.

The corrector's resistance means the tolerance for errors occurring during identification (incorrect model structure or approximation of the transmittance of the control object) or for changes in the object's parameters (amplification factor, time constants, delay) during its operation.
It is desirable that even if the mathematical model of the object adopted in the design process is not correct, the control system is stable and its regulation is close to optimal.

Therefore, in order to test the resistance of the corrector, it is necessary to design the corrector for an object with transmittance $G_0(s)$ (as described in points 3a, 3b and 4) and then check its operation in a situation where the actual transmittance of the object $G'_0(s)$ differs from the one adopted
in the design process, see Fig. 3.

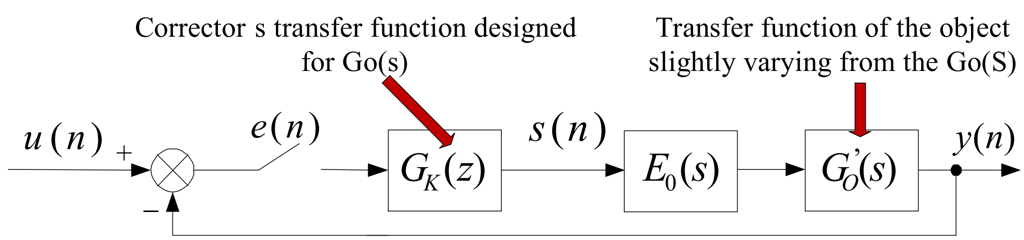

Fig. 3. The system diagram shows the resistance analysis of the designed correctors.

4. Useful commands.

When designing correctors, you can use the following commands available in Matlab:

c2dm
feedback
series
zgrid
ginput
help